Math 1B
Quiz 5 Version
Thu Nov 1, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ____________________

SCORE: _____ / 30 POINTS

NO CALCULATORS ALLOWED

Find the length of the curve $y = x^{\frac{3}{2}} - \frac{1}{3}x^{\frac{1}{2}}$ from $x = 1$ to $x = 4$. **SCORE: _____ / 6 PTS**

Your final answer must be a number.

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{6}x^{-\frac{1}{2}}$ IS CONTINUOUS ON $[1,4]$

$$\underline{\int_1^4 \sqrt{1 + \left(\tfrac{3}{2}x^{\frac{1}{2}} - \tfrac{1}{6}x^{-\frac{1}{2}}\right)^2}\,dx} \ (2)$$

$$= \int_1^4 \sqrt{1 + \tfrac{9}{4}x - \tfrac{1}{2} + \tfrac{1}{36}x^{-1}}\,dx$$

$$= \underline{\int_1^4 \sqrt{\tfrac{9}{4}x + \tfrac{1}{2} + \tfrac{1}{36}x^{-1}}\,dx} \ (1)$$

$$= \underline{\int_1^4 \left(\tfrac{3}{2}x^{\frac{1}{2}} + \tfrac{1}{6}x^{-\frac{1}{2}}\right)dx} \ (1)$$

$$(1) = \underline{\left(x^{\frac{3}{2}} + \tfrac{1}{3}x^{\frac{1}{2}}\right)}\Big|_1^4$$

$$= 4^{\frac{3}{2}} - 1^{\frac{3}{2}} + \tfrac{1}{3}\left(4^{\frac{1}{2}} - 1^{\frac{1}{2}}\right)$$

$$= 8 - 1 + \tfrac{1}{3}(2-1)$$

$$= \underline{7\tfrac{1}{3} \text{ or } \tfrac{22}{3}}$$

(1)

For the function $f(x) = x^2 - 8x$ on the interval $[1, 4]$, find the value of c guaranteed by the Integral Mean Value Theorem (ie. such that $f(c) = f_{ave}$). **NOTE: The final answer is irrational.** **SCORE: _____ / 8 PTS**

$$f(c) = \underline{\frac{1}{4-1}\int_1^4 (x^2 - 8x)\,dx} \ (2)$$

$$c^2 - 8c = \frac{1}{3}\underline{\left(\tfrac{1}{3}x^3 - 4x^2\right)}\Big|_1^4$$

$(1\frac{1}{2})$

$$c^2 - 8c = \frac{1}{3}\left(\tfrac{1}{3}(64-1) - 4(16-1)\right)$$

$$c^2 - 8c = \frac{1}{3}(21 - 60)$$

$$\underline{c^2 - 8c} = \underline{-13} \ (2)$$

(1)

$$c^2 - 8c + 13 = 0$$

$$c = \frac{8 \pm \sqrt{64 - 52}}{2} = \frac{8 \pm 2\sqrt{3}}{2} = 4 \pm \sqrt{3}$$

SUBTRACT 1 POINT IF YOU INCLUDED BOTH $4+\sqrt{3}$ AND $4-\sqrt{3}$ IN FINAL ANSWER

$$(1\tfrac{1}{2}) \ \underline{c = 4 - \sqrt{3}} \in [1,4]$$

Find the length of the parametric curve $\begin{array}{l} x = 3t^3 - 4t \\ y = 2 - 6t^2 \end{array}$ for $1 \le t \le 2$. **SCORE: _____ / 6 PTS**

Your final answer must be a number.

$\frac{dx}{dt} = 9t^2 - 4 \qquad \frac{dy}{dt} = -12t$

$\int_1^2 \sqrt{(9t^2-4)^2 + (-12t)^2}\, dt$ 2

$= \int_1^2 \sqrt{81t^4 - 72t^2 + 16 + 144t^2}\, dt$

$= \int_1^2 \sqrt{81t^4 + 72t^2 + 16}\, dt$ 1

$= \int_1^2 (9t^2 + 4)\, dt$ 1

1

$= (3t^3 + 4t)\Big|_1^2$

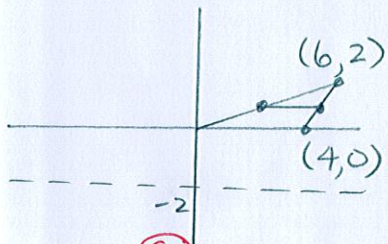
$= 3(8-1) + 4(2-1)$

$= 25$

1

The region bounded by $y = \frac{1}{3}x$, $y = x - 4$ and $y = 0$ is revolved around $y = -2$. **SCORE: _____ / 10 PTS**

Find the volume of the resulting solid using calculus. **Your final answer must be a number.**



1

$\frac{1}{3}x = x - 4$

$x = 3x - 12$

$12 = 2x$

$x = 6$

$y = \frac{1}{3}x \to x = 3y$

$y = x - 4 \to x = y + 4$

1 $2\pi \int_0^2 (y - {}^-2)(y + 4 - 3y)\, dy$

1 2 1

$= 2\pi \int_0^2 (y+2)(4-2y)\, dy$

2

$= 2\pi \int_0^2 (8 - 2y^2)\, dy$

$= 2\pi \left(8y - \frac{2}{3}y^3\right)\Big|_0^2$

1

$= 2\pi \left(8(2) - \frac{2}{3}(8)\right)$

$= 2\pi \left(\frac{32}{3}\right)$

$= \frac{64\pi}{3}$ 1